A Rømer time-delay determination of the gravitational-wave propagation speed

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Abstract

In 1676 Olaus Rømer presented the first observational evidence for a finite light velocity $c_{\rm em}$. He formed his estimate by attributing the periodically varying discrepancy between the observed and expected occultation times of the Galilean satellite Io by its planetary host Jupiter to the time it takes light to cross Earth's orbital diameter. Given a stable celestial clock that can be observed in gravitational waves the same principle can be used to measure the propagation speed $c_{\rm gw}$ of gravitational radiation. Space-based "LISA"-like detectors will, and terrestrial LIGO-like detectors may, observe such clocks and thus be capable of directly measuring the propagation velocity of gravitational waves. In the case of space-based detectors the clocks will be galactic close white dwarf binary systems; in the case of terrestrial detectors, the most likely candidate clock is the periodic gravitational radiation from a rapidly rotating non-axisymmetric neutron star. Here we evaluate the accuracy that may be expected of such a Rømer-type measurement of $c_{\rm gw}$ by foreseeable future space-based and terrestrial detectors. For space-based, LISA-like detectors, periodic sources are plentiful: by the end of the first year of scientific operations a LISA-like detector will have measured $c_{\rm gw}$ to better than a part in a thousand. Periodic sources may not be accessible in terrestrial detectors available to us in the foreseeable future; however, if such a source is detected then with a year of observations we could measure $c_{\rm gw}$ to better than a part in a million.

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I. INTRODUCTION

Over the course of a Jovian synodic year the distance light must transit between Earth and Jupiter varies by two astronomical units: approximately 3×10^8 km. If we neglect the time required for light to cross Earth's orbit, the interval between events that are periodic at Jupiter will at Earth be observed at times that may vary from the expected by as much as 10^3 s. It was this observed variation between the observed and expected occultations of the Galilean satellite Io that led Olaus Rømer to conclude that light has a finite propagation speed and to the first real measurement of light's velocity c_{em} [1, 2]. Galactic compact white dwarf binary systems, or rapidly rotating non-axisymmetric neutron stars, are similarly regular clocks whose periodic signal is transmitted to Earth via gravitational radiation. In the same way that Rømer was able to use observations of the discrepancies in the optically measured times of Io's occultations by Jupiter to measure the speed of light, so we may use gravitational-wave observations by space-based LISA-like detectors [3–7] of compact white dwarf binary systems, or by terrestrial LIGO-like detectors [8–11] of rapidly rotating neutron stars, to measure the propagation speed c_{gw} of gravitational waves.

In general relativity theory gravitational waves and light wave both propagate on spacetime null geodesics; correspondingly, there is no difference in their respective (vacuum) propagation speeds. A direct measurement of the gravitational-wave propagation speed is, thus, a "go/no-go" test of the theory.

Measurement of the Rømer delay directly and unambiguously access the wave propagation speed across Earth's orbital baseline. This stands in contrast to other proposed tests of general relativity whose results are sometimes discussed in terms of the gravitational-wave propagation speed, but whose interpretation in this way requires a theoretical model or phenomenological framework to relate the observation to the propagation speed. For example, Will [12] suggested searching for an anomalous (compared to general relativity's prediction) compression of the gravitational-wave signal from an inspiraling binary system. Such a compression could be interpreted as a frequency-dependent gravitational-wave propagation speed resulting from a non-zero "graviton-mass". In a similar vein, Larson and Hiscock [13], Cutler et al. [14] proposed measuring the phase difference between the binary's orbital phase at some fiducial time as determined optically and by gravitational-wave observations. The phase difference, relative to that predicted by general relativity, could then be inter-

preted as differences in the gravitational-wave propagation speed at a frequency twice the binary orbital frequency. What is relevant is that, unlike the measurement described in this work, none of these other measurements directly accesses the gravitational-wave propagation speed: i.e., their interpretation in terms of the wave propagation speed requires a theory or phenomenological framework that relates the observed phenomena to the wave propagation speed. As shown by Carlip [15] in the context of a recent claim to have measured the propagation speed of the gravitational force [16] through its effect on light travel, change the theory or framework and the interpretation changes.

In Section II we estimate the precision to which we can measure the gravitational-wave propagation speed from multi-year observations of periodic gravitational waves. We assume here that the gravitational-wave frequency and sky location of the source are known *a priori*. For such cases, we show that the Fisher matrix estimate of the uncertainty can be expressed very simply in terms of the source parameters and orbital radius of the Earth's motion around the Sun, valid for *all* ground-based and proposed space-based detectors. Details specific to the detectors, such as antenna pattern functions, cancel out when the uncertainty is expressed in terms of the signal-to-noise ratio of the measurement. We also discuss the complications introduced if we relax the assumption of *a priori* knowledge of the source frequency and sky location of the gravitational-wave source. Although the calculation is more complicated for this case, the Fisher matrix formalism can still be used to estimate the uncertainty in the gravitational-wave propagation speed as a function of the source parameters and detector geometry. In Section III we use the general result of Section II to obtain numerical values for "3σ" fractional uncertainties in $c_{\rm gw}$ for observations in terrestrial and space-based detectors. We also summarize our conclusions.

II. ESTIMATING THE MEASUREMENT PRECISION OF THE GRAVITATIONAL-WAVE PROPAGATION SPEED

A. Introduction

We use the Fisher Information Matrix formalism [17, 18] to estimate the precision with which we can estimate the gravitational-wave propagation speed from multi-year observations of periodic gravitational waves whose frequency and propagation direction are known

a priori. For example, in the case of observations in a ground-based detector or detector network, the source may be a rapidly rotating neutron star whose rotational frequency and sky location are known from observation of its radio pulses; or, in the case of observations made with a space-based detector, the source may be a close white dwarf binary system that has also been observed optically.

For any monocrhomatic source we may write the TT gauge gravitational wave strain at time t and location $\vec{x}$ as

$$\boldsymbol{h}(t,\vec{x}) = h_+(t,\vec{x})\boldsymbol{e}_+ + h_\times(t,\vec{x})\boldsymbol{e}_\times \,, \tag{1a}$$

where $\boldsymbol{e}_+$ and $\boldsymbol{e}_\times$ are orthogonal polarization tensors, fixed in inertial space, and

$$h_+(t,\vec{x}) = H_+ \cos\left[2\pi f_{\text{gw}} u + \Phi_+\right] \,, \tag{1b}$$

$$h_\times(t,\vec{x}) = H_\times \cos\left[2\pi f_{\text{gw}} u + \Phi_\times\right] \,, \tag{1c}$$

$$u = t - \frac{\hat{k}\cdot\vec{x}}{c_{\text{em}}(1+\epsilon)} \,. \tag{1d}$$

Here ϵ is the fractional difference between light speed (c_{em}) and the gravitational-wave propagation speed ($c_{\text{gw}} = c_{\text{em}}(1+\epsilon)$), $\hat{k}$ is the wave-propagation direction and H_+, $H_\times$, Φ_+ and $\Phi_\times$ are constants determined by the source orientation with respect to $\boldsymbol{e}_+$ and $\boldsymbol{e}_\times$.

For detectors that are small compared to the observed radiation wavelength[23] we may write the detector response to the incident $(h_+, h_\times)$ as

$$r(t) = F_+(t)h_+(t,\vec{x}(t)) + F_\times(t)h_\times(t,\vec{x}(t)) \,, \tag{2}$$

where $\vec{x}(t)$ is the detector location. The functions F_+ and $F_\times$ are determined by the projection of the detector's antenna pattern on the wave polarization tensors. Both terrestrial and space-based LISA-like detectors are constantly changing their orientation with respect to $\boldsymbol{e}_+$ and $\boldsymbol{e}_\times$ (in the case of terrestrial detectors owing to Earth's diurnal motion about its rotation axis, and in the case of space-based detectors owing to the science-craft constellation's annual motion about about Sol); correspondingly, F_+ and $F_\times$ are time dependent.

B. Known source location and frequency

The Fisher Information matrix $\mathcal{I}$ has elements

$$\mathcal{I}_{jk}(\vec{\theta}) = \frac{2}{\sigma_n^2}\int_0^T \frac{\partial r}{\partial\theta_j}\frac{\partial r}{\partial\theta_k} dt \,, \tag{3}$$

where T is the observation duration, σ_n^2 is the detector noise power spectral density at the gravitational wave frequency $f_{\rm gw}$, and $\vec{\theta}$ denotes the parameter vector $(\epsilon, H_+, H_\times, \Phi_+, \Phi_\times)$. The partial derivatives of r with respect our parameterization are

$$\frac{\partial r}{\partial \epsilon} = -2\pi f_{\rm gw} \frac{\hat{k}\cdot\vec{x}}{c_{\rm em}(1+\epsilon)^2} \left\{F_+ H_+ \sin\left[2\pi f_{\rm gw} u + \Phi_+\right] + F_\times H_\times \sin\left[2\pi f_{\rm gw} u + \Phi_\times\right]\right\} \,, \tag{4a}$$

$$\frac{\partial r}{\partial H_+} = F_+ \cos\left(2\pi f_{\rm gw} u + \Phi_+\right) \,, \tag{4b}$$

$$\frac{\partial r}{\partial H_\times} = F_\times \cos\left(2\pi f_{\rm gw} u + \Phi_\times\right) \,, \tag{4c}$$

$$\frac{\partial r}{\partial \Phi_+} = -F_+ H_+ \sin\left(2\pi f_{\rm gw} u + \Phi_+\right) \,, \tag{4d}$$

$$\frac{\partial r}{\partial \Phi_\times} = -F_\times H_\times \sin\left(2\pi f_{\rm gw} u + \Phi_\times\right) \,. \tag{4e}$$

For all cases of interest the gravitational-wave detectors follow Earth's orbit about Sol; correspondingly,

$$\hat{k}\cdot\vec{x} = (R_{\rm au}\cos\theta)\cos(\omega_\odot t - \phi)\,, \tag{5}$$

where $R_{\rm au}$ is Earth's orbital radius (1 au), $\omega_\odot$ is the detector angular velocity in its orbital motion about Sol (2π/yr), θ is the ecliptic latitude, and ϕ is the azimuthal angle of the source with respect to Earth's orbital position at $t = 0$. (See Figure 1.) (The small displacement $\vec{d}$ of a terrestrial detector away from the Earth's orbital path about Sol introduces an order $d/R_{\rm au} \sim 0.25\%$ correction, which we ignore.)

To evaluate the components of the Fisher matrix we take advantage of the sinusoidal periodicity of F_+, $F_\times$, h_+ and $h_\times$ and focus on observations that are integer multiples of a year duration. Noting that

$$\omega_\odot \ll 2\pi f_{\rm gw} \ll c_{\rm em}/d\,, \tag{6}$$

$$\omega_\odot \ll c_{\rm em}/R_{\rm au}\,, \tag{7}$$

the integrals for the Fisher matrix elements $\mathcal{I}_{\epsilon j}$ for $T > 1\,\text{yr}$ quickly simplify to

$$\mathcal{I}_{\epsilon\epsilon} = \frac{(2\pi f_{\rm gw} R_{\rm au}\rho\cos\theta)^2}{c_{\rm em}^2(1+\epsilon)^4}\,, \tag{8}$$

$$\mathcal{I}_{\epsilon j} = 0 \qquad \text{for } j \in \{H_+, H_\times, \Phi_+, \Phi_\times\}\,, \tag{9}$$

where ρ^2 is the (power) signal-to-noise ratio

$$\rho^2 = \frac{1}{2\sigma_n^2}\int_0^T r^2(t)dt\,. \tag{10}$$

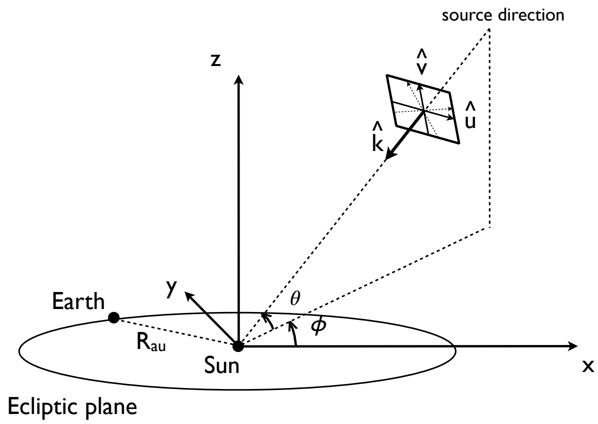

FIG. 1: The relevant geometric quantities used in the calculation: $\hat{k}$ is the unit vector pointing in the direction of wave propagation; θ is the ecliptic latitude (i.e., the angle that $-\hat{k}$ makes with the plane of the ecliptic); ϕ is the azimuthal angle of the source with respect to the Earth's orbital position at $t = 0$. The detector antenna pattern functions F_+ and $F_\times$ from Eq. (2) are defined with respective to the polarization tensors constructed from $\hat{u}$ and $\hat{v}$, which are proportional to the unit vectors $\hat{\phi}$ and $\hat{\theta}$, respectively.

Correspondingly, at the level of the Cramer-Rao bound there is no co-variance between the uncertainty in our estimate of ϵ and any of the other problem parameters. The expected variance of the estimate for ϵ is thus

$$\nu_\epsilon = \left(\mathcal{I}^{-1}\right)_{\epsilon\epsilon} \tag{11}$$

$$= \frac{1}{\mathcal{I}_{\epsilon\epsilon}} = \left(\frac{c_{\mathrm{em}}}{2\pi f_{\mathrm{gw}} R_{\mathrm{au}}}\right)^2 \frac{(1+\epsilon)^4 \sec^2\theta}{\rho^2}\,. \tag{12}$$

This result is valid for observations in all ground-based detectors and all proposed space-based detectors, whether Earth- or solar-orbiting. It is also valid for detector arrays where the data are combined coherently as described in, e.g., [19, 20]. (In the case of detector arrays ρ^2 is the *array* power signal-to-noise ratio.) Details specific to the detectors, such as the antenna pattern functions F_+ and $F_\times$, cancel out when the uncertainty is expressed in terms of the signal-to-noise ρ.

C. Unknown source location and/or frequency

If the source frequency, sky location, or both are not known *a priori* one needs to enlarge the parameter vector $\vec{\theta}$ to include the additional unknowns: e.g., the frequency $f_{\rm gw}$ and/or the source location on the sky $(\theta,\ \phi)$. The Fisher matrix dimensionality thus expands to include terms involving partial derivatives

$$\frac{\partial r}{\partial f_{\rm gw}} = -2\pi\left(t - \frac{\hat{k}\cdot\vec{x}}{c_{\rm em}(1+\epsilon)}\right)\left\{F_+H_+\sin\left[2\pi f_{\rm gw}u+\Phi_+\right] + F_\times H_\times \sin\left[2\pi f_{\rm gw}u+\Phi_\times\right]\right\}\ , \tag{13}$$

$$\begin{aligned}\frac{\partial r}{\partial \theta} &= \frac{2\pi f_{\rm gw}}{c_{\rm em}(1+\epsilon)}\frac{\partial(\hat{k}\cdot\vec{x})}{\partial\theta}\left\{F_+H_+\sin\left[2\pi f_{\rm gw}u+\Phi_+\right] + F_\times H_\times \sin\left[2\pi f_{\rm gw}u+\Phi_\times\right]\right\}\\ &\quad + \left\{\frac{\partial F_+}{\partial\theta}H_+\cos\left[2\pi f_{\rm gw}u+\Phi_+\right] + \frac{\partial F_\times}{\partial\theta}H_\times\cos\left[2\pi f_{\rm gw}u+\Phi_\times\right]\right\}\ ,\end{aligned} \tag{14}$$

$$\begin{aligned}\frac{\partial r}{\partial \phi} &= \frac{2\pi f_{\rm gw}}{c_{\rm em}(1+\epsilon)}\frac{\partial(\hat{k}\cdot\vec{x})}{\partial\phi}\left\{F_+H_+\sin\left[2\pi f_{\rm gw}u+\Phi_+\right] + F_\times H_\times \sin\left[2\pi f_{\rm gw}u+\Phi_\times\right]\right\}\\ &\quad + \left\{\frac{\partial F_+}{\partial\phi}H_+\cos\left[2\pi f_{\rm gw}u+\Phi_+\right] + \frac{\partial F_\times}{\partial\phi}H_\times\cos\left[2\pi f_{\rm gw}u+\Phi_\times\right]\right\}\ ,\end{aligned} \tag{15}$$

where

$$\frac{\partial(\hat{k}\cdot\vec{x})}{\partial\theta} = -R_{\rm au}\sin\theta\,\cos(\omega_\odot t-\phi)\,, \tag{16}$$

$$\frac{\partial(\hat{k}\cdot\vec{x})}{\partial\phi} = +R_{\rm au}\cos\theta\,\sin(\omega_\odot t-\phi)\,. \tag{17}$$

Comparing these expressions with the partial derivative $\partial r/\partial\epsilon$ from Equation 4a, one can see that the off-diagonal Fisher matrix elements $\mathcal{I}_{\epsilon j}$ for $j\in\{f_{\rm gw},\theta,\phi\}$ are *non-zero*. Correspondingly, the elements of the covariance matrix $(\mathcal{I}^{-1})_{\epsilon k}$ are no longer trivial and ν_ϵ no longer simply expressed. How well we can estimate the propagation speed of gravitational waves using observations of periodic sources whose frequency or sky location are not known *a priori* is the subject of work in-progress.

III. DISCUSSION

As described here, to measure the gravitational-wave propagation speed from the Rømer delay it is necessary to monitor a periodic source of gravitational waves, whose position on the sky and radiation frequency is known, for a year or more. For terrestrial detectors

such a source might be a radio pulsar that also radiates gravitationally. For such sources, equation 11 may be written as

$$\nu_\epsilon = \left(3.2\times 10^{-7}\right)^2 \left(\frac{100\,\mathrm{Hz}}{f_{\mathrm{gw}}}\right)^2 \left(\frac{10}{\rho}\right)^2 (1+\epsilon)^4 \sec^2\theta; \tag{18}$$

i.e., the "3σ" fractional uncertainty in the measurement of the gravitational-wave propagation speed arising from a one or more year observation of a 100 Hz gravitational-wave source situated on the ecliptic plane is $10^{-6}(10/\rho)$. Since a signal-to-noise $\rho \simeq 10$ is typically taken as the threshold for source detection in a ground-based detector or detector network, if a periodic source is observed a measurement of c_{gw} to 3σ precision, $300(10/\rho)\,\mathrm{m\ s^{-1}}$, will follow shortly.

There are no reliable predictions for the gravitational-wave amplitude associated with rapidly rotating neutron stars. Mass asymmetries — "mountains" — are limited in size by the tensile strength of the neutron star crust, while the potential for fluid circulation instabilities (r-modes) to lead to gravitational-wave emission depends on the (temperature dependent) neutron star surface "ocean" shear and bulk viscosities [21, §7.3]. It may well be the case that neutron star crusts cannot support a sufficiently large asymmetry to be observable gravitational-wave sources, or that the neutron star fluid viscosity is always so great as to stabilize neutron star fluid r-modes. Likewise, it may be that circumstances can be contrived that lead neutron stars to be strong radiation sources for ground-based detectors, but that there is no natural mechanism for creating or placing the neutron star into such states. Thus, while a sensitive measurement of the gravitational-wave speed is possible with ground-based detectors, carrying it out depends on the observation of a type of source that may not be available to us.

Strong sources of periodic gravitational waves, in the form of galactic white dwarf binary systems, are both certain and plentiful for any of the proposed "LISA"-like space-based gravitational wave detectors [3]. For such sources, Equation 11 is conveniently written as

$$\nu_\epsilon = \left(3.2\times 10^{-4}\right)^2 \left(\frac{10\,\mathrm{mHz}}{f_{\mathrm{gw}}}\right)^2 \left(\frac{100}{\rho}\right)^2 (1+\epsilon)^4 \sec^2\theta. \tag{19}$$

An amplitude signal-to-noise of 100 in a one-year observation is the minimum expected for a typical "verification binary" in a space-based detector; correspondingly, the "3σ" fractional uncertainty in the measurement of the gravitational-wave propagation speed arising from a one or more year observation of a 10 mHz gravitational-wave source situated on the ecliptic plane is a quite respectable $10^{-3}(100/\rho)$.

At present, then, we find ourselves in an odd position. With the observation of a periodic gravitational-wave source, we know how to make a direct, accurate and unambiguous measure of the gravitational-wave propagation speed and, from it, a "go/no-go" test of general relativity theory. On the one hand, for existing or foreseeable future ground-based detectors there is no guarantee that there will, or — indeed — can, exist any source that will enable the measurement. On the other hand, there are scores sources, already identified, that are accessible to a space-based detector that would enable such a measurement but, despite the strong recommendation of the United States National Research Council [22], NASA abandoned its committment to the decade-long ESA/NASA partnership that would have led to the construction of such an observatory and no such project is currently planned by either agency. We can only hope that the feasibility of accurately and unambiguously testing general relativity — by means such as described here — will strengthen the case for reviving a LISA-like gravitational wave observatory in the near future.

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[23] True for all terrestrial detectors. For LISA-like detectors, true for wave frequency $f \lesssim 10$ mHz.